

TRANSPORTATION SYMPOSIUM

2019

FRP-RC Design - Part 2

Steve Nolan

Adapted from...

Composites Australia, December 5, 2018

Design of concrete structures internally reinforced with FRP bars

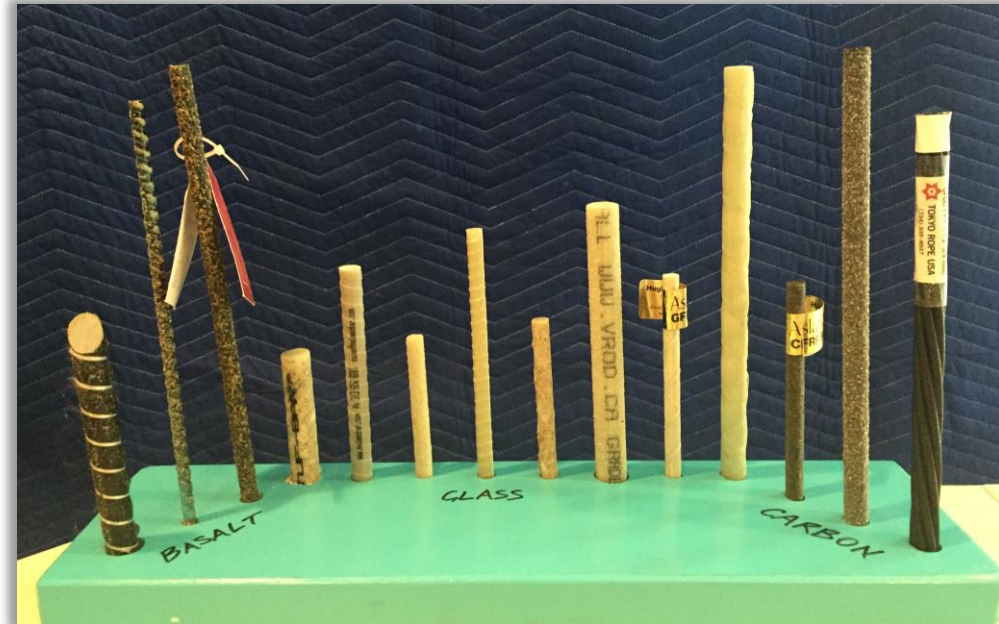
**Canada Research Chair in Advanced Composite Materials for Civil Structures
NSERC/Industrial Research Chair in Innovative FRP Reinforcement for Concrete
Director, The University of Sherbrooke Research Centre on FRP Composites
Department of Civil Engineering**

University of Sherbrooke, Sherbrooke, QC, Canada

[E-mail:brahim.benmokrane@usherbrooke.ca](mailto:brahim.benmokrane@usherbrooke.ca)

Course Description

Fiber-reinforced polymer (FRP) materials have emerged as an alternative for producing reinforcing bars for concrete structures. Due to other differences in the physical and mechanical behavior of FRP materials versus steel, unique guidance on the engineering and construction of concrete structures reinforced with FRP bars is necessary.



Learning Objectives

- Understand the mechanical properties of FRP bars
- Describe the behavior of FRP bars
- Describe the design assumptions
- Describe the flexural/shear/compression design procedures of concrete members internally reinforced with FRP bars
- Describe the use of internal FRP bars for serviceability & durability design including long-term deflection
- Review the procedure for determining the development and splice length of FRP bars.

Content of the Course

FRP-RC Design - Part 1, (50 min.)

This session will introduce concepts for reinforced concrete design with FRP rebar. Topics will address:

- Recent developments and applications
- Different bar and fiber types;
- Design and construction resources;
- Standards and policies;

FRP-RC Design - Part 2, (50 min.)

This session will introduce Basalt FRP rebar that is being standardized under FHWA funded project **STIC-0004-00A** with extended FDOT research under BE694, and provide training on the flexural design of beams, slabs, and columns for:

- Design Assumptions and Material Properties
- Ultimate capacity and rebar development length under strength limit states;
- Crack width, sustained load resistance, and deflection under service limit state;

Content of the Course

BFRP-RC Design - Part III, (50 min.)

This session continues with Basalt FRP rebar from Part II, covering shear and axial design of columns at the strength limit states for:

- Fatigue resistance under the Fatigue limit state;
- Shear resistance of beams and slabs;
- Axial Resistance of columns;
- Combined axial and flexure loading.

FRP-RC Design - Part IV *(Not included at FTS - for future training):*

This session continues with FRP rebar from Part III, covering detailing and plans preparation:

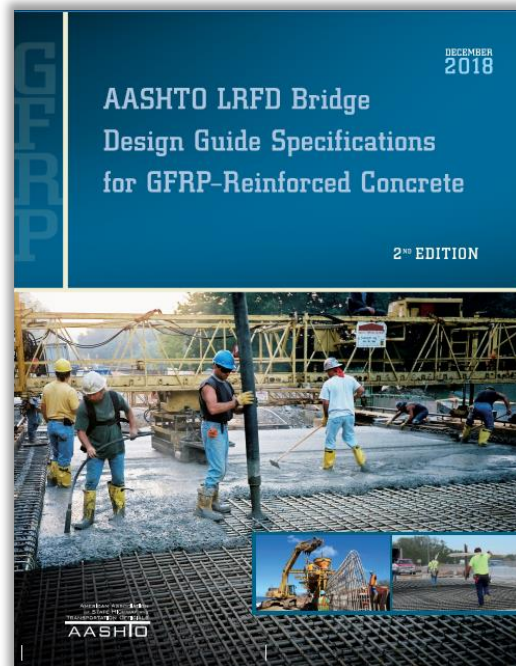
- Minimum Shrinkage and Temperature Reinforcing
- Bar Bends and Splicing
- Reinforcing Bar Lists
- General Notes & Specifications

Session 2: Design Assumptions and Material Properties

- FRP bar is anisotropic
 - High strength only in the fiber direction
 - Anisotropic behavior affects shear strength, dowel action and bond performance
- FRP bar does not exhibit yielding: is elastic until failure
 - Design accounts for lack of ductility

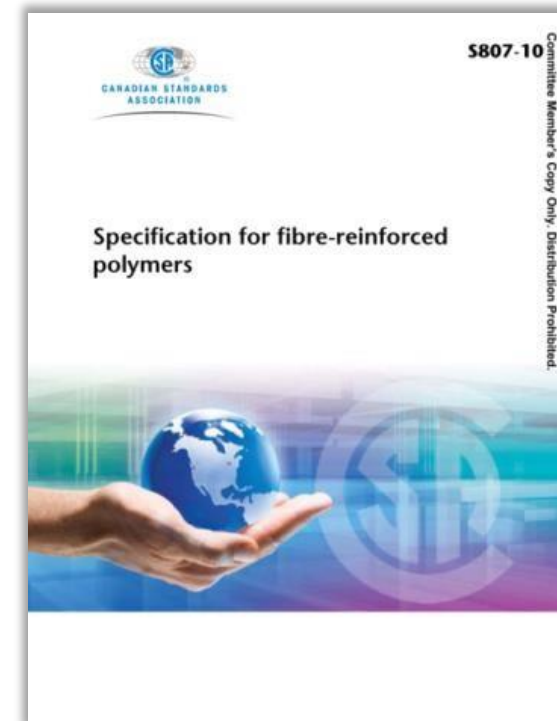
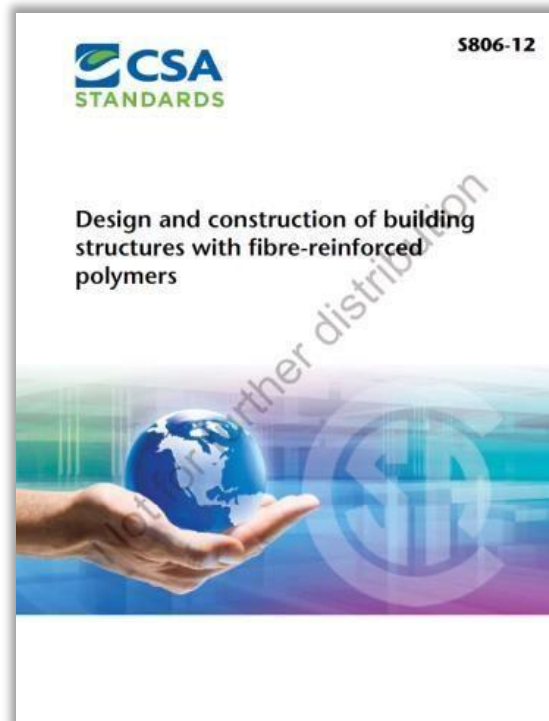
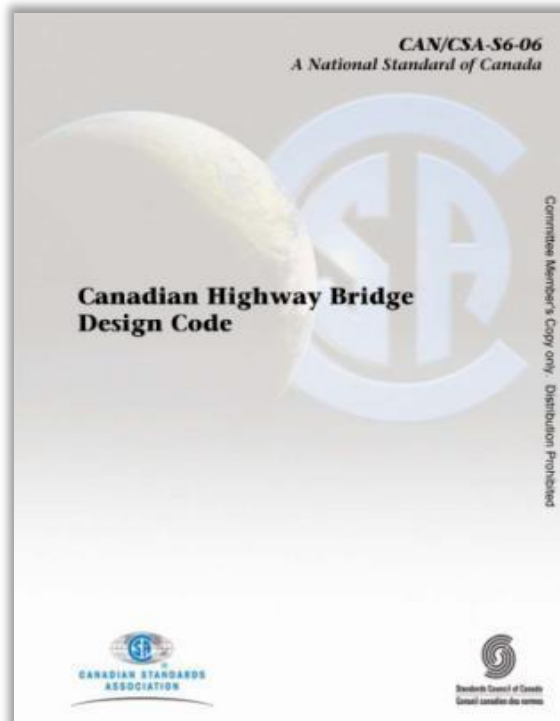
Session 2: Design Assumptions and Material Properties

1. **AASHTO-GS2:** “LRFD Bridge Design Guide Specifications for GFRP-Reinforced Concrete Bridges *2nd Edition* (2018)
2. **FDOT Construction and Materials Specifications:** Section 932-3 “Fiber Reinforced Polymer (FRP) Reinforcing Bars” (2019) (*BFRP-2020*)
3. **ASTM D7957-17:** “Standard Specification for Solid Round Glass Fiber Reinforced Polymer Bars for Concrete Reinforcement“. *1st Edition* (2017)



Session 2: Design Assumptions and Material Properties

1. **CAN/CSA S6:** Canadian Highway Bridge Design Code, Section 16 "Fibre Reinforced Polymers (FRP) Structures".
2. **CAN/CSA S806:** Design and Construction of Building Components with FRP.
3. **CAN/CSA-S807:** Specifications for Fibre Reinforced Polymers.



Session 2: Design Assumptions and Material Properties

General Standard Philosophy

These are limit states-based standards

- They follow the same basic procedures as other **AASHTO or CSA** structural design standards (concrete structures reinforced with steel bars)
- They are intended primarily for design of concrete structures reinforced internally with FRP bars and/or grids or externally with sheets and laminates (for repair and retrofit)
- The standards cover areas for which adequate theoretical and experimental evidence is available to justify the relevant clauses
- The standards are intended to be self-contained
- The design provisions are intended to be on the conservative side

Session 2: Design Assumptions and Materials

Load Factors and Load Combinations

- **AASHTO-GS2** uses the same load factors as in **AASHTO-LRFD BDS**
- **CSA S806** uses the same load factors as in **CSA A23.3-14** code. Load combinations are also the same as in **CSA A23.3-14**, which are based on the National Building Code of Canada
- **CSA S6 – Section 16** on FRP Structures - uses the same load factors and load combinations as in the **Section 8** on Concrete Structures of the **CHBDC**

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FRP Bar Sizes:

- **FDOT 932-3 and ASTM D7957**
- **CSA S807 similar**

Table 3-1 Sizes and Tensile Loads of FRP Reinforcing Bars						
Bar Size Designation	Nominal Bar Diameter (in)	Nominal Cross Sectional Area (in ²)	Measured Cross-Sectional Area (in ²)		Minimum Guaranteed Tensile Load (kips)	
			Minimum	Maximum	BFRP and GFRP Bars	CFRP Bars
2	0.250	0.049	0.046	0.085	6.1	10.3
3	0.375	0.11	0.104	0.161	13.2	20.9
4	0.500	0.20	0.185	0.263	21.6	33.3
5	0.625	0.31	0.288	0.388	29.1	49.1
6	0.750	0.44	0.415	0.539	40.9	70.7
7	0.875	0.60	0.565	0.713	54.1	-
8	1.000	0.79	0.738	0.913	66.8	-
9	1.128	1.00	0.934	1.137	82.0	-
10	1.270	1.27	1.154	1.385	98.2	-

TABLE 3 Geometric and Mechanical

Bar Designation No.	Nominal Dimensions		Minimum	Maximum	Tensile Force kN [kip]
	Diameter mm [in.]	Cross-Sectional Area mm ² [in. ²]			
M6 [2]	6.3 [0.250]	32 [0.049]	30 [0.046]	55 [0.085]	27 [6.1]
M10 [3]	9.5 [0.375]	71 [0.11]	67 [0.104]	104 [0.161]	59 [13.2]
M13 [4]	12.7 [0.500]	129 [0.20]	119 [0.185]	169 [0.263]	96 [21.6]
M16 [5]	15.9 [0.625]	199 [0.31]	186 [0.288]	251 [0.388]	130 [29.1]
M19 [6]	19.1 [0.750]	284 [0.44]	268 [0.415]	347 [0.539]	182 [40.9]
M22 [7]	22.2 [0.875]	387 [0.60]	365 [0.565]	460 [0.713]	241 [54.1]
M25 [8]	25.4 [1.000]	510 [0.79]	476 [0.738]	589 [0.913]	297 [66.8]
M29 [9]	28.7 [1.128]	645 [1.00]	603 [0.934]	733 [1.137]	365 [82.0]
M32 [10]	32.3 [1.270]	819 [1.27]	744 [1.154]	894 [1.385]	437 [98.2]

Session 2: Design Assumptions and Materials

Mechanical Properties and Behavior

Tensile Behavior

- The guaranteed (characteristic or specific) tensile strength for FRP reinforcement shall be the mean tensile strength minus three times the standard deviation (**ASTM D7957, CSA-S806 & -S6**)
- Similarly, the guaranteed (characteristic or specific) rupture tensile strain of FRP reinforcement shall be the mean rupture tensile strain minus three times the standard deviation (**ASTM D7957, CSA-S806 & -S6**)
- Similarly, the design elastic modulus for FRP reinforcement shall be the mean modulus (**ASTM D7957, CSA-S806 & -S6**).

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Mechanical Properties and Behavior

Tensile Strength & Modulus of Elasticity of GFRP Bars

- Tensile strength ranges between 77 to 250 ksi (530 to 1700 MPa); **FDOT 932-3** range 77 to 124 minimum.
- Modulus of elasticity ranges between 5,800 to 9,500 ksi (40 to 65 GPa); **FDOT 932-3** minimum 6,500 ksi.

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Mechanical Properties and Behavior

CAN CSA S807-10 – Grades of FRP Bars

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Specification for fibre-reinforced polymers

Table 2
Grades of FRP bars and grids corresponding to their
minimum modulus of elasticity, GPa
(See [Clause 8.3](#) and [Table 3](#))

Designation	Grade I		Grade II		Grade III	
	Individual bars	Bars in a grid	Individual bars	Bars in a grid	Individual bars	Bars in a grid
AFRP	50	40	70	60	90	80
CFRP	80	70	110	100	140	130
GFRP	40	30	50	40	60	50

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Mechanical Properties and Behavior

CAN CSA S807-19 *(Public Review Draft)* – **Grades of FRP Straight Bars**

Table 2A
**Grades of FRP straight bars and grids corresponding to their
minimum modulus of elasticity, GPa**

(See [Clauses 8.1.1, 8.3 and 10.1](#), and [Table 3](#))

Designation	Grade I		Grade II		Grade III	
	Individual bars	Bars in a grid	Individual bars	Bars in a grid	Individual bars	Bars in a grid
AFRP	50	40	70	60	90	80
BFRP	50	40	60	50	70	60
CFRP	80	70	110	100	140	130
GFRP	40	30	50	40	60	50

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Mechanical Properties and Behavior

CAN CSA S807-19 (*Public Review Draft*) – Grades of FRP Bent Bars

Table 2B

Grades of FRP bent bars corresponding to their minimum modulus of elasticity of the straight portion, GPa

(See [Clauses 8.1.1, 8.3](#) and [10.1](#), and [Table 3](#))

Designation	Grade IB	Grade IIB	Grade IIIB
	Individual bars	Individual bars	Individual bars
AFRP	50	60	65
BFRP	50	55	60
CFRP	80	100	120
GFRP	40	45	50

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Mechanical Properties and Behavior

Tensile Properties of V-ROD GFRP bars of **Grade I**
(ISIS Canada Manual No. 3)

Lowest

Metric size	Nominal diameter (mm)	Nominal Area (mm ²)	Tensile modulus of elasticity (MPa)	Guaranteed tensile strength (MPa)
#3	10	71	42500 (6,164 ksi)	899
#4	13	129	44100	825
#5	15	199	42500	800
#6	20	284	44500	733
#8	25	510	43900	654 (95 ksi)

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Mechanical Properties and Behavior

Tensile properties of V-ROD GFRP bars of **Grade II**
(ISIS Canada Manual No. 3)

Medium

Metric size	Nominal diameter (mm)	Nominal Area (mm ²)	Tensile modulus of elasticity (MPa)	Guaranteed tensile strength (MPa)
#3	10	71	52500	1200
#4	13	129	53400	1161
#5	15	199	53600	1005
#6	20	284	55400	930
#7	22	387	56600	882
#8	25	510	53500	811
#10	32	819	52900	776

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Mechanical Properties and Behavior

Tensile properties of V-ROD GFRP bars of **Grade III**
(ISIS Canada Manual No. 3)

Highest

Metric size	Nominal diameter (mm)	Nominal Area (mm ²)	Tensile modulus of elasticity (MPa)	Guaranteed tensile strength (MPa)
#3	10	71	65100	1734 (251 ksi)
#4	13	129	65600	1377
#5	15	199	62600	1239
#6	20	284	64700	1196
#7	22	387	62600	1005
#8	25	510	66400 (9,630 ksi)	1064
#10	32	819	65100	1105

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Tensile Strength of the FRP at Bend

FRP bars can be fabricated with bends, however the tensile strength is reduced by 40 – 50%

AASHTO-GS2, *CSA S6 and CSA S806 Codes*

$$f_{fb} = \left(0.05 \left(\frac{r_b}{d_b} \right) + 0.3 \right) f_{fu} \leq f_{fu}$$

r_b is the bend radius

d_b is the diameter of reinforcing bar

f_{fu} is the design tensile strength of FRP

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Compression Behavior of the FRP

FRP compression reinforcement is considered in **AASHTO-GS2**, **CSA S806** and **CSA S6 (New Edition, 2019)**

For the purpose of design, assume zero compression strength and stiffness.

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Bond Behavior

- Bond strength is a function of:
 - 1) The bar design and surface roughness
 - 2) Mechanical properties of the bar itself
- Bond Force can be transmitted by:
 - 1) Adhesion resistance at bar interface (chemical bond)
 - 2) Frictional resistance of interface (friction bond)
 - 3) Mechanical interlock due to surface irregularity
- Adequate cover is essential
- f_c' of the concrete affects bond.
- In general the bond of FRP bars to concrete is equivalent to the bond of steel bars

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Time Dependent Behavior – Creep Rupture

- FRP reinforcing bars subjected to a sustained load over time can fail after a time period called the **endurance time**.
- This phenomenon is known as **creep rupture** (or **static fatigue**)
- The higher the stress, the shorter the lifetime
- GFRP → BFRP → AFRP → CFRP (least susceptible)

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Time Dependent Behavior – Creep Rupture

The maximum stress in FRP bars or grids under loads at serviceability limit state shall not exceed the following fraction of the guaranteed tensile strength **AASHTO-GS2** (*CSA-S806 & CSA-S6*):

- AFRP : **n/a** (0.35)
- CFRP : **0.65** (0.65)
- GFRP : **0.30** (0.25)

(The maximum strain in GFRP tension reinforcement under sustained service loads shall not exceed 0.002).

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Durability Design

- One of the chief benefits of FRP bars
- FRP bars do not rust, but are susceptible in degrees to high pH (BFRP & GFRP) or moisture (AFRP)
- Depends on type of fiber, resin used, quality of manufacturing, degree of cure, etc.

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Durability Design

Example of Durability Related Provisions (*CSA-S807*):

1. Limit on Constituent Material, e.g.
 - Limits on diluents and certain fillers
 - Limits on low-profile additives
 - No blended resins
2. Lower Limit on Glass Transition Temperature (T_g) & Cure Ratio
 - Minimum cure ratio and T_g
3. Material Screening Through Physical & Durability Properties
 - Maximum void content
 - Maximum water absorption
 - Limits on mechanical property loss in different environment conditioning (Alkali)

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Durability Design

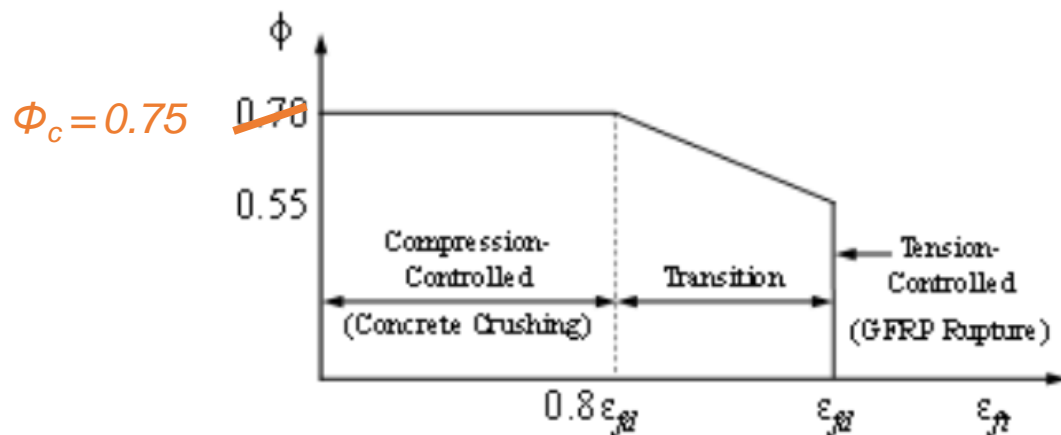
The **AASHTO-GS2**, **CSA-S806** and **-S6** address the durability issue in design of FRP reinforced sections through a common way considering the following:

- The material resistance & environmental reduction factors based on fiber type and exposure conditions
- Limitation of maximum stress under service load
- Limitation of maximum crack-width under service load
- Limitation of maximum stress/strain level under sustained load
- Concrete cover (fire resistance)
- Creep rupture stress limits
- Fatigue stress limits
- Factor for long-term deflection calculation

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Resistance factors (*AASHTO-GS2*, *CSA S806*)

- For non-prestressed FRP reinforcement, the resistance factor, Φ_F , shall be taken as $\Phi_F = 0.75$ (compression-controlled only in *AASHTO-GS2*; For tension-controlled $\Phi_F = 0.55$)
- Concrete and steel resistance factors remain the same.



2.5.5.2—Resistance Factors

The resistance factor, ϕ , shall be taken as:

- For compression-controlled and tension-controlled reinforced concrete sections as specified in Article 2.6.3:

$$\phi = \begin{cases} 0.55 & \text{for } \epsilon_{ft} = \epsilon_{fd} \\ 1.55 - \frac{\epsilon_{ft}}{\epsilon_{fd}} & \text{for } 0.80\epsilon_{fd} < \epsilon_{ft} < \epsilon_{fd} \\ 0.75 & \text{for } \epsilon_{ft} \leq 0.80\epsilon_{fd} \end{cases} \quad (2.5.5.2-1)$$

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Material Resistance Factors

Resistance factors from AASHTO-GS2		
Material	Notation	Factor
Concrete-cast-in-situ	ϕ_c	0.75
Concrete-precast	ϕ_c	0.75
Steel reinforcement	ϕ_s	0.90
CFRP (PC)	ϕ_f	(0.75)
AFRP	ϕ_f	n/a
GFRP	ϕ_f	0.55

Resistance factors from CSA S806		
Material	Notation	Factor
Concrete-cast-in-situ	ϕ_c	0.65
Concrete-precast	ϕ_c	0.70
Steel reinforcement	ϕ_s	0.85
CFRP	ϕ_f	0.75
AFRP	ϕ_f	0.75
GFRP	ϕ_f	0.75

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Material Resistance Factors

Application (<i>CSA-S6</i>)	Resistance Factor ϕ_{FRP}
AFRP reinforcement in concrete and NSMR	0.65
AFRP in externally-bonded applications	0.55
AFRP and aramid fibre rope tendons for concrete and timber	0.60
CFRP reinforcement in concrete	0.80
CFRP in externally-bonded applications and NSMR	0.80
CFRP tendons	0.80
GFRP reinforcement in concrete	0.55
GFRP in externally-bonded applications and NSMR	0.70
GFRP tendons for concrete components	0.55
GFRP tendons for timber decks	0.70

Application (<i>AASHTO</i>)	Resistance Factor C_E
CFRP tendons embedded (PC) (<i>AASHTO-CFRP-GS</i>)	1.00
CFRP tendons external (PT) (<i>AASHTO-CFRP-GS</i>)	0.90
GFRP reinforcement in concrete (interior) (<i>AASHTO-GS2</i>)	0.80
GFRP reinforcement in concrete (exterior) (<i>AASHTO-GS2</i>)	0.70

Session 2: Design Assumptions and Materials

Serviceability of FRP Reinforced Concrete Members (Beams & Slabs)

- Deflections under service loads often control design
- Designing FRP RC beams or slabs for concrete crushing satisfies serviceability criteria for deflections and crack width
- Cracking and deflections are defined as:
 - Cracking — Excessive crack width is undesirable for aesthetic and other reasons (for example, to prevent water leakage) that can damage or deteriorate the structural concrete
 - Deflection — Deflections should be within acceptable limits imposed by the use of the structure (for example, supporting attached nonstructural elements without damage).
- The substitution of FRP for steel on an equal area basis would typically result in larger deflections and wider crack widths

Session 2: Design Assumptions and Material Properties

Crack control: (CSA-S806)

The crack control parameter, z :

$$z = k_b \frac{E_s}{E_f} f_f \sqrt[3]{d_c A}$$

$z < 45\,000$ N/mm for interior exposure and $38\,000$ N/mm for exterior exposure. (Sections satisfied the $38\,000$ N/mm criterion)

$f_f < 0.25 f_{fu}$ for GFRP bars.

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Crack Control Reinforcement (**CSA-S6**)

When the maximum tensile strain in FRP reinforcement under service loads exceeds 0.0015, cross-sections of the component in maximum positive and negative moment regions shall be so proportioned that the crack-width must not exceed 0.5 mm (0.02-in.) for member subject to aggressive environments otherwise 0.7 mm (0.028-in.), where the crack width is given by:

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

- The value of k_b shall be determined experimentally, but in the absence of test data it may be taken as;
 - 0.8 for sand-coated
 - and 1.0 for deformed FRP bars.
- In calculating d_c , the clear cover shall not be taken greater than 50 mm (2.0-in.)

Crack Control Reinforcement (CSA S6)

- Check crack widths when tensile strain in FRP at SLS exceeds 0.0015 (stress of 60 MPa in Grade 1) which is almost always the case.
- Maximum permitted crack widths:
 - $w_{cr} \leq 0.50$ mm for members subject to aggressive environments
 - $w_{cr} \leq 0.70$ mm for members with other exposures
- Maximum permitted crack widths are double what is permitted for reinforcing steel since GFRP does not corrode.
- Crack width derived from an analytical model:

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

Crack Control Reinforcement (CSA S6)

Maximum crack width to be checked against the limit

Factor to calculate the maximum crack width (1.5 for mean width, 1 for minimum width)

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

$f_{FRP}/E_{FRP} = \epsilon_{FRP}$
is the average strain in FRP reinforcement

Term k_b to account for the bond of the bar to concrete:

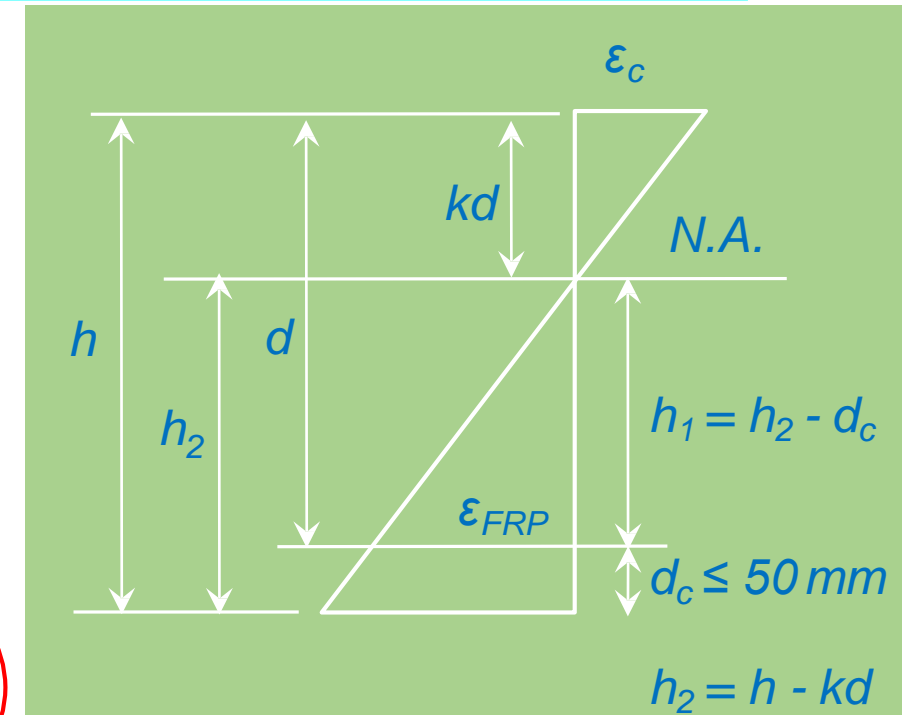
- use $k_b = 0.8$ for both sand-coated and 1.0 for deformed bars

Crack Control Reinforcement (CSA S6)

Geometrical relationship which accounts for the amplification of the average strain from the FRP bar to the exposed surface of the concrete

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

Effect of the bar spacing and bar cover on the basic crack width



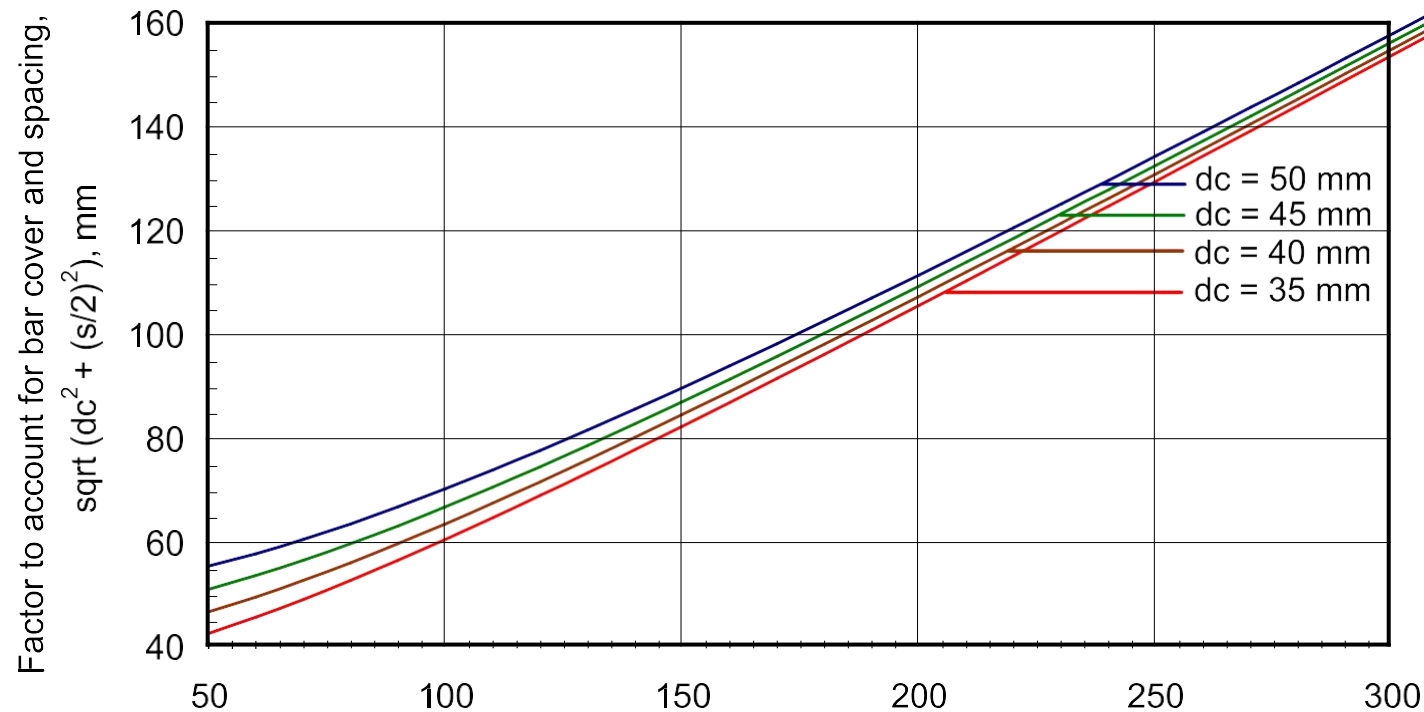
s is the bar spacing

d_c is the distance from the *centroid* of tension GFRP to the extreme tension surface of the concrete ≤ 50 mm

Effect of Bar Spacing and Cover on Crack-Width,

W_{cr}

$$w_{cr} = 2 \frac{f_{FRP}}{E_{FRP}} \frac{h_2}{h_1} k_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$



Session 2: Design Assumptions and Material Properties

Deflection Limits of FRP Reinforced Concrete Members (CSA S806)

S806-02

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Table 11
Maximum Permissible Computed Deflections
(See Clause 8.3.2.1.)

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to specified live load, L	$\ell_n/180^*$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to specified live load, L	$\ell_n/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of the nonstructural elements (sum of the long-time deflection due to all sustained loads and the immediate deflection due to any additional live load) \ddagger	$\ell_n/480^\dagger$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell_n/240§$

*Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and consideration of long-time effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

† Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

\ddagger Long-time deflections shall be determined in accordance with Clause 8.3.2.4 but may be reduced by the amount of deflection calculated to occur before the attachment of nonstructural elements.

$§$ Not to be greater than the tolerance provided for nonstructural elements. Limiting deflection may be exceeded if camber is provided so that the total deflection minus camber does not exceed the limit shown in this Table.

Session 2: Design Assumptions and Material Properties

Deflection Calculation (CSA-S806)

- Deflection shall be calculated based on moment-curvature (M/EI) relationship
- Integrate M/EI relationship or use moment-area method

$$\delta_A = \int_0^L \frac{mM}{EI} dx$$

Session 2: Design Assumptions and Material Properties

Deflection Calculation (CSA S806)

- Effective moment of inertia (used for Direct Method):
 - When a section is uncracked, its moment of inertia is equal to the gross moment of inertia, I_g
 - When the applied moment, M_a , exceeds the cracking moment, M_{cr} , cracking occurs, which causes a reduction in the stiffness and the moment of inertia is based on the cracked section, I_{cr}

$$I_g = bh^3/12 \text{ as before}$$

- Using n_f as the modular ratio between the FRP reinforcement and the concrete

$$n_f = E_{FRP} / E_C$$

Where:

$$I_{cr} = \frac{b d^3}{3} k^3 + n_f A_f d^2 (1-k)^2$$

$$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f$$

Session 2: Design Assumptions and Material Properties

Long-Term Deflection

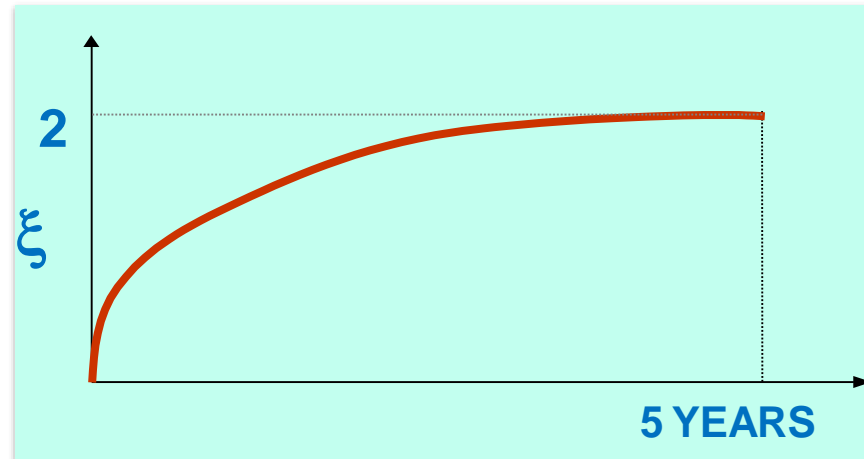
Long-term deflection under sustained load

The total immediate plus long-term deflection for flexural members shall be obtained by multiplying the immediate deflections caused by the sustained load considered by the factor $[1+S]$.

- $S = 2.0$ for 5 years or more
- $S = 1.5$ for 12 months
- $S = 1.3$ for 6 months
- $S = 1.1$ for 3 months

Session 2: Design Assumptions and Material Properties

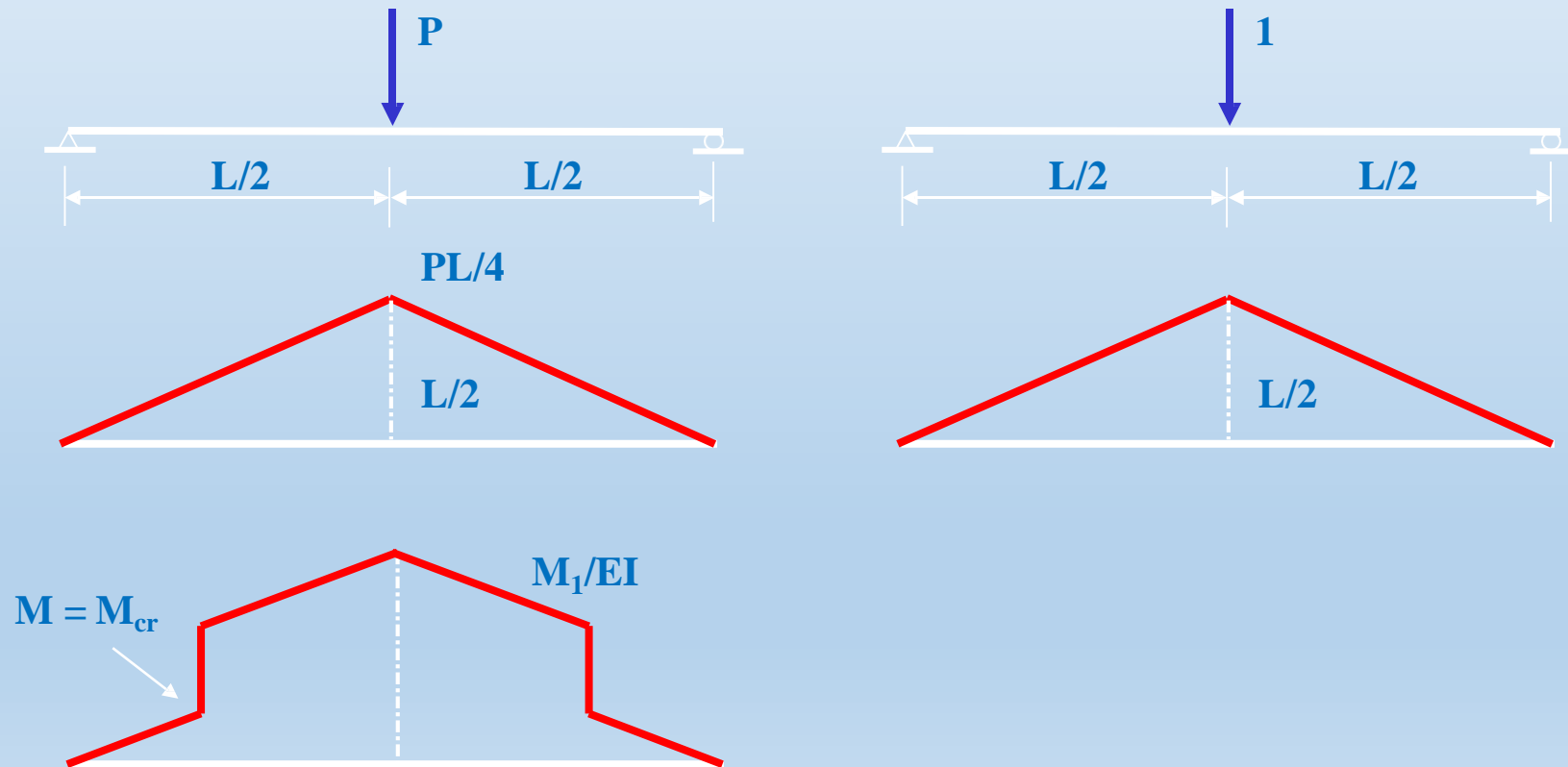
Long-Term Deflection



For same design strength long term deflection is 3 – 4 times greater than members reinforced by steel reinforcement

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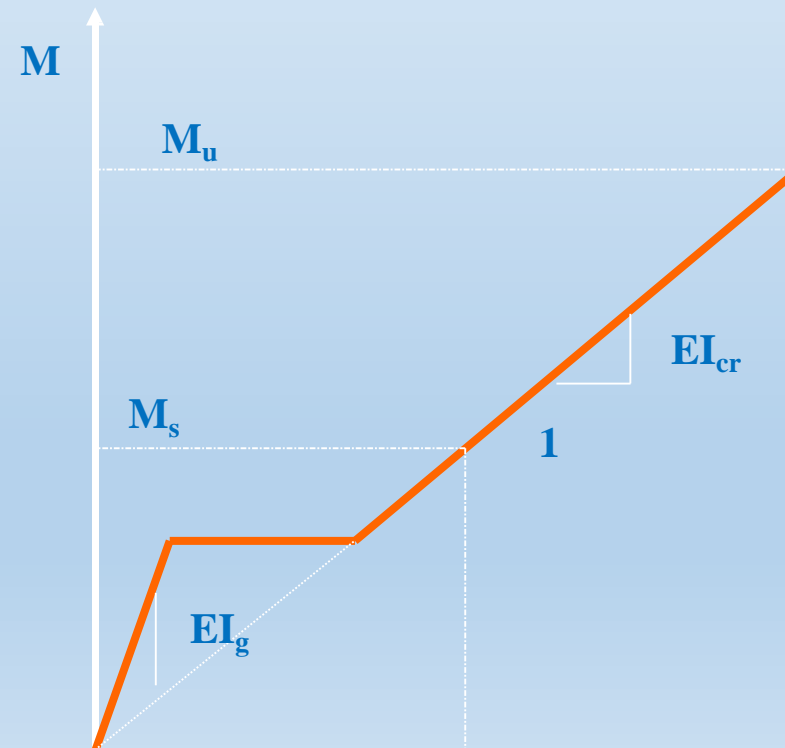
Deflection Calculation



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Deflection Calculation

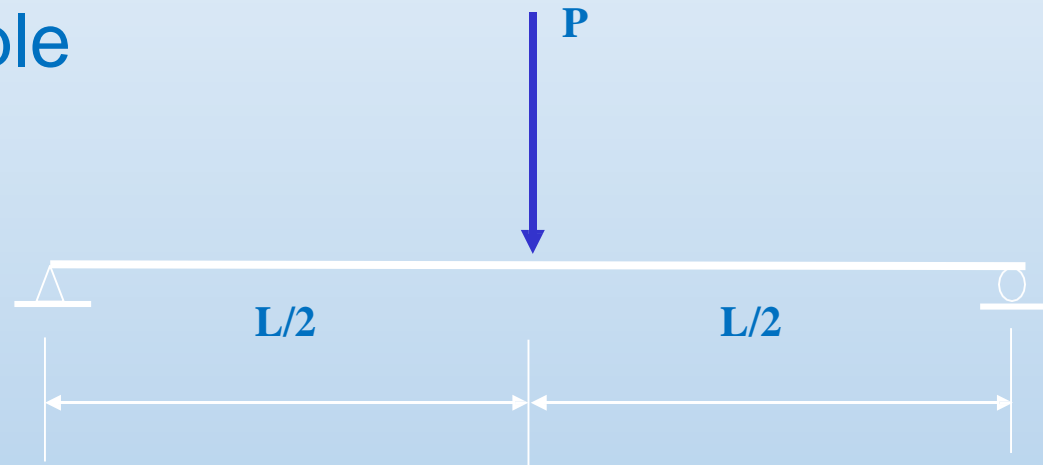
- Moment-Curvature Relationship for FRP Reinforced Section



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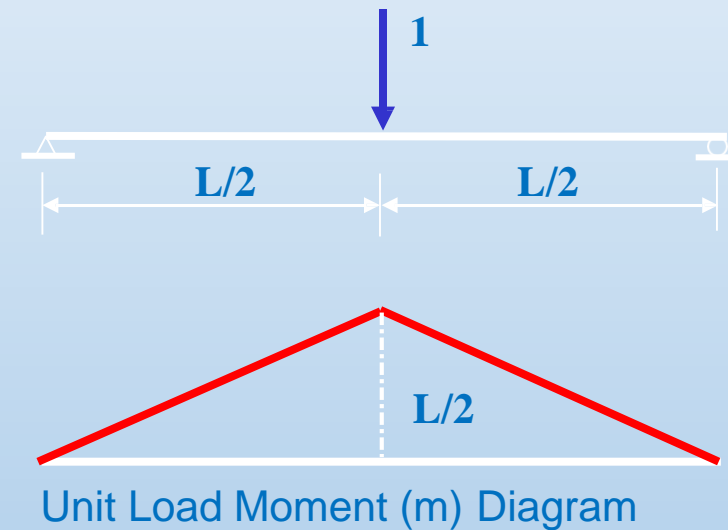
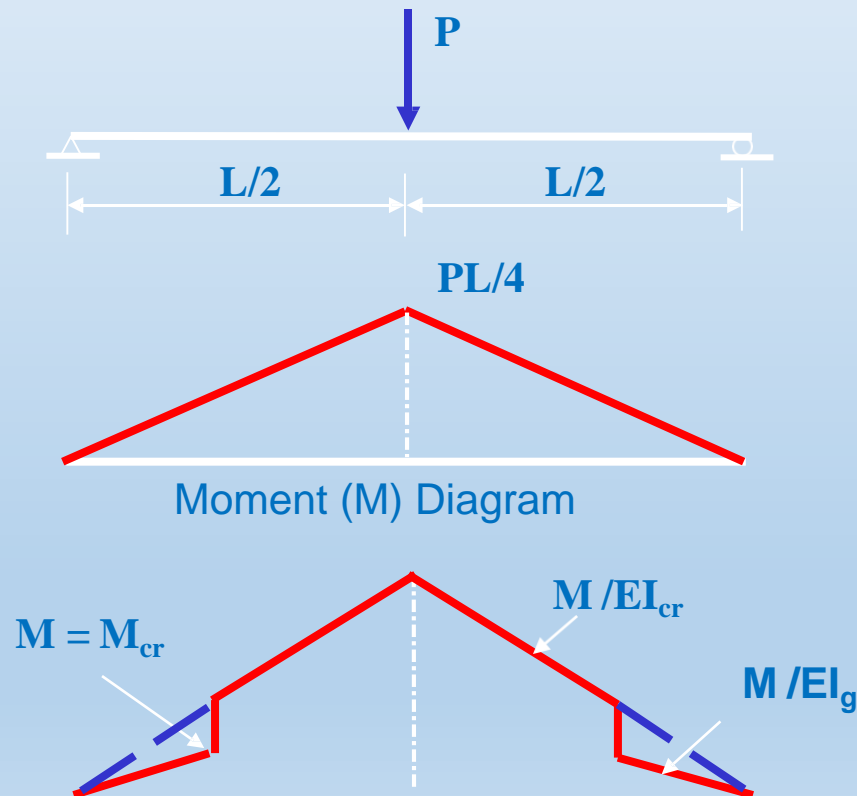
Deflection Calculation

- Example



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Deflection Calculation



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Deflection Calculation Maximum Deflection

$$\delta_{\max} = \delta_{cr} - \delta_c$$

Deflection of the fully cracked beam

Correction from
the uncracked
sections

From Regular Strength of Materials

$$\delta_{cr} = \frac{PL^3}{48E_c I_{cr}}$$

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Deflection Calculation

Correction Term

$$\delta_c = 2 \int_0^{L_g} m_1 \eta \frac{M}{EI_{cr}} dx$$

\approx

$$\delta_c = \frac{PL^3}{48E_c I_{cr}} 8\eta \left(\frac{L_g}{L} \right)^3$$

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Deflection Calculation

Maximum Deflection

$$\delta_{\max} = \frac{PL^3}{48E_c I_{cr}} \left[1 - 8\eta \left(\frac{L_g}{L} \right)^3 \right]$$

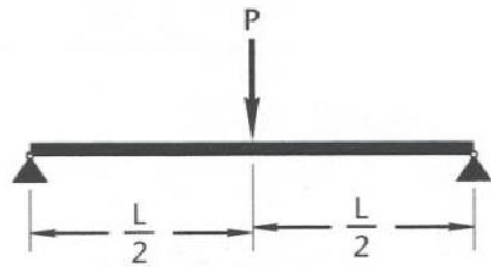
Where:

$$\eta = \left(1 - \frac{I_{cr}}{I_g} \right)$$

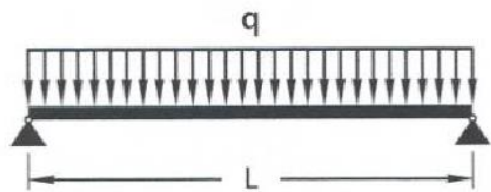
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Deflection Calculation

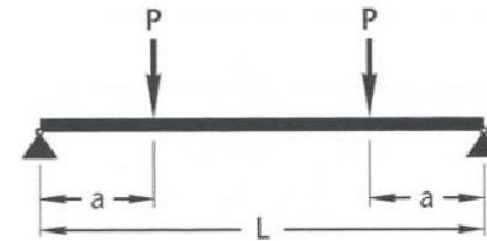
Maximum Deflection of Other Load Cases



$$\delta_{\max} = \frac{PL^3}{48 E_c I_{cr}} \left[1 - 8\eta \left(\frac{L_g}{L} \right)^3 \right]$$



$$\delta_{\max} = \frac{5qL^4}{384 E_c I_{cr}} \left[1 - \frac{192}{5} \eta \left[\frac{1}{3} \left(\frac{L_g}{L} \right)^3 - \frac{1}{4} \left(\frac{L_g}{L} \right)^4 \right] \right]$$



$$\delta_{\max} = \frac{PL^3}{24 E_c I_{cr}} \left[3 \left(\frac{a}{L} \right) - 4 \left(\frac{a}{L} \right)^3 - 8\eta \left(\frac{L_g}{L} \right)^3 \right]$$

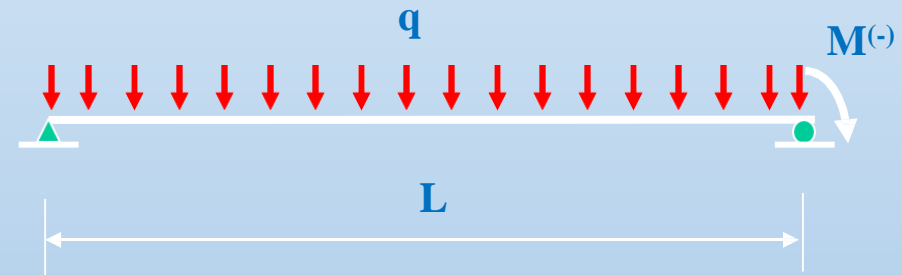
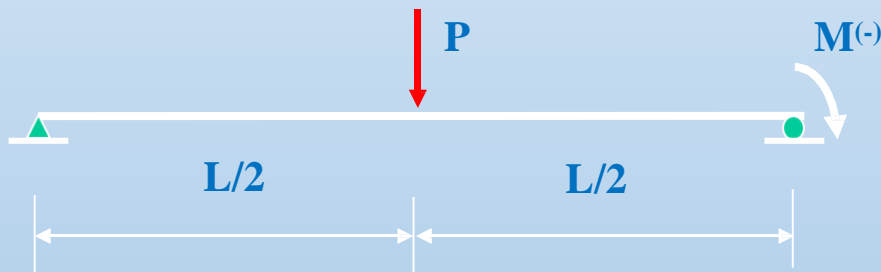
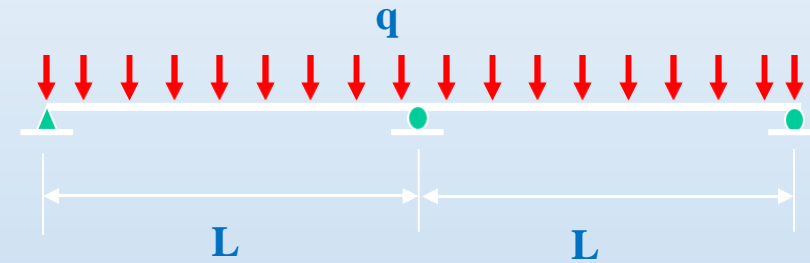
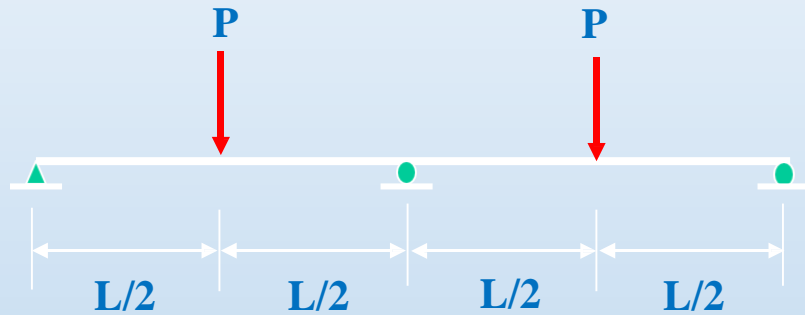


$$\delta_{\max} = \frac{PL^3}{3 E_c I_{cr}} \left[1 - \eta \left(\frac{L_g}{L} \right)^3 \right]$$

Note: $\eta = \left(1 - \frac{l_{cr}}{l_g} \right)$

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Deflection of Continuous Beams



$$\delta_{\max} = \frac{PL^3}{48E_c I_{cr}} \left[\frac{5}{16} - \frac{15}{8} \eta \left(\frac{L_g}{L} \right)^3 \right]$$

$$\delta_{\max} = \frac{5qL^4}{384E_c I_{cr}} \left[\frac{3}{5} - \frac{36}{10} \eta \left(\frac{L_g}{L} \right)^3 \right]$$

Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

Development length of FRP bars in tension l_d shall be taken:

$$l_d = 1.15 \frac{k_1 k_2 k_3 k_4 k_5}{d_{cs}} \frac{f_F}{\sqrt{f'_c}} A_b \geq 300mm$$

k_1 = bar location factor

k_2 = concrete density factor

k_3 = bar size factor

k_4 = bar fibre factor

k_5 = bar surface profile factor

d_{cs} = smaller of :

1. distance from closest concrete surface to the center of the bar
2. two-thirds of the center-to-center spacing d_{cs}

shall not be greater than $2.5 d_b$

Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

Development length of FRP bars in tension

The maximum permissible value of $(f'_c)^{0.5}$ shall be 5 MPa

Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

Modification factors:

k_1 (Bar location factor) :1.3; 1.0

k_2 (Concrete density factor) :1.3; 1.2; 1.0

k_3 (Bar size factor) :0.8; 1.0

k_4 (Bar fibre factor) :1.0; 1.25

k_5 (Bar surface profile) :1.0; 1.05; 1.80

Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

Development of bent bar:

$$165k_2 \frac{d_b}{\sqrt{f'_c}} \text{ for } f_F \leq 520 \text{ MPa}$$

$$\frac{f_F}{3.1} k_2 \frac{d_b}{\sqrt{f'_c}} \text{ for } 520 \leq f_F \leq 1040 \text{ MPa}$$

$$330k_2 \frac{d_b}{\sqrt{f'_c}} \text{ for } f_F > 1040 \text{ MPa}$$

- L_d not less than $12d_b$ or 230 mm
- The tail length of a bent bar, l_t , should not be less than $12d_b$
- The bend radius, r_b , should not be less than $3d_b$

Session 2: Design Assumptions and Material Properties

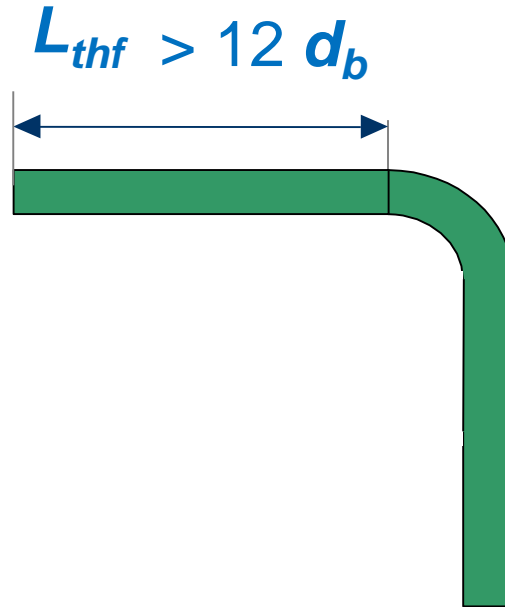
Development Length and Splice of Reinforcement (CSA-S806)

Anchorage of shear reinforcement:

- Web reinforcement shall be carried as close to the compression and tension surfaces of a member as practically feasible. **(Clause 9.9.1)**
- Unless it is determined that the shear reinforcement can develop its design strength at mid-height of the beam or column cross-section, FRP web reinforcement shall consist of closed loops or spiral reinforcement.
- The web reinforcement shall have sufficient development length to develop its design stress at mid-height of the member.

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Detailing of shear stirrups



L_{thf} = length of tail beyond a hook

Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA S806)

Splices of reinforcement:

- The lap splice length shall be $1.3l_d$, where l_d is the basic development length of the bar (**Clause 9.10.3**);
- Lap splices of bundled bars shall be based on the lap splice length required for individual bars within a bundle, increased by 20% for a two-bar bundle and 30% for a three-bar bundle. Individual bar splices within a bundle shall not overlap (**Clause 9.10.4**);
- Spliced bars in flexural members shall have a transverse spacing not exceeding the lesser of one-fifth of the required lap splice length or 130mm (**Clause 9.10.5**).

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Development Length and Splice of Reinforcement (CSA S806)

Mechanical anchorage– Clause:

- Mechanical anchorage including headed bars or headed studs may be used, provided their effectiveness has been demonstrated by tests that closely simulate the condition in the field and that they can develop at least 1.67 times the required design strength.



Session 2: Design Assumptions and Material Properties

Development Length and Splice of Reinforcement (CSA-S6)

The development length, l_d , of FRP bars in tension shall be calculated from:

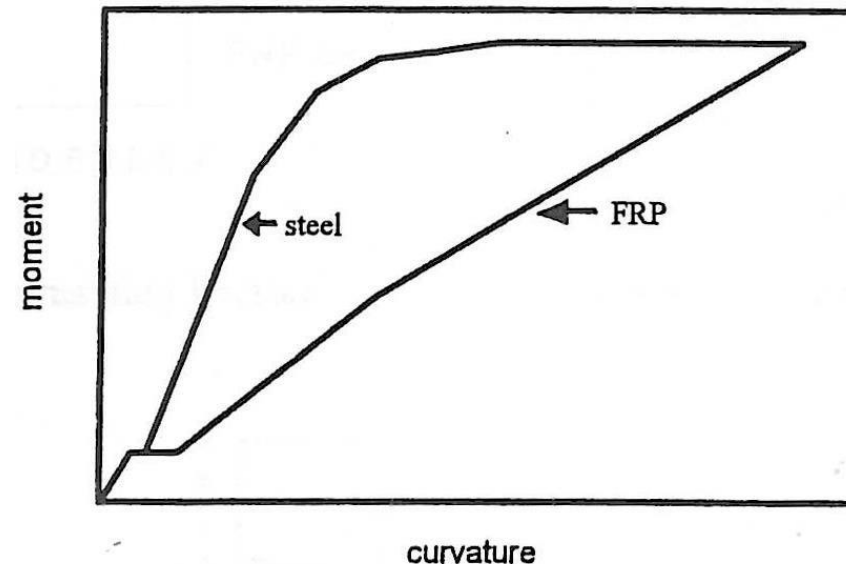
$$l_d = 0.45 \frac{k_1 k_4}{\left(d_{cs} + K_{tr} \frac{E_{FRP}}{E_s} \right)} \left(\frac{f_{FRPu}}{f_{cr}} \right) A$$

The splice length for FRP bars in tension shall be $1.3l_d$.

Spliced FRP bars shall not be separated by more than 150 mm.

Deformability (*CSA-S6*)

- Deformability takes into account absorbed energy based on deformability, to ensure adequate deformation of members reinforced with FRP.
- The purpose of calculating deformability in an FRP reinforced section is to provide a comparable deformability as expected of a comparably reinforced steel reinforced section.



(Jaeger et al. 1997)

Deformability (*CSA-S6*)

- Overall performance factor, J , must be at least 4.0 for rectangular sections and 6.0 for T-sections.

$$J = \frac{M_{ult} \psi_{ult}}{M_c \psi_c}$$

- M_{ult} and ψ_{ult} are moment and curvature at ultimate limit state.
 - M_{ult} = moment at ultimate limit state
 - $\psi_{ult} = \epsilon_{ult} / kd$
- M_c and ψ_c are moment and curvature corresponding to a concrete strain of 0.001.
 - $M_c = f_c k (1-k/3) bd^2$
 - $\psi_c = \epsilon_c / kd$, where $\epsilon_c = 0.001$
- Use $f_c = \epsilon_c E_c$ or $f_c = 1.8 f'_c (\epsilon_c / \epsilon'_c) / (1 + (\epsilon_c / \epsilon'_c)^2)$

Deformability (*CSA-S6*)

- For calculating M_{ult} and Ψ_{ult} , repeat the same steps as required to calculate M_r , but with higher resistance factors.
- For calculating M_{ult} and Ψ_{ult} use the following:
 - $\Phi_c = 1.00$
 - $\Phi_{FRP} = 1.00$
- Based on the definition of J as a function of M_c , tension-controlled members may not have adequate deformability.
- Deformability may govern the design of deep members or T-beam members (i.e. pier caps or diaphragms).

How Does GFRP Compare with Steel?

- For typical reinforcement ratios found in a bridge deck slab, factored moment resistance at ULS with GFRP and reinforcing steel is similar (within 30%).
- If the member is subjected to SLS moment:
 - ❑ SLS will govern the design of a member with GFRP reinforcement. ULS usually governs the design of a member reinforced with steel.
 - ❑ On average, a GFRP design will require 50% to 100% more reinforcement as a design with reinforcing steel.
 - ❑ Use the smallest practical bar diameter and bar spacing for an efficient design with GFRP (less of a consideration for design with reinforcing steel).
 - ❑ Avoid bar spacing of less than 100 mm to avoid congestion of bars.

Questions

Co-presenters:

Raphael Kampmann PhD

*FAMU-FSU College of Engineering
Tallahassee, FL.*

kampmann@eng.famu.fsu.edu

Marco Rossini, PhD student

*University of Miami.
Coral Gables, FL.*

mxr1465@mami.edu

FDOT Design Contacts:

Steven Nolan, P.E.

*FDOT State Structures Design Office,
Tallahassee, FL.*

Steven.Nolan@dot.state.fl.us

FDOT Materials and manufacturing:

Chase Knight, Ph.D, P.E.

*State Materials Office,
Gainesville, FL.*

Chase.Knight@dot.state.fl.us